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Density stratified swirl flows; an improved turbulence model

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The k-ε model of turbulence was modified to allow for the effect of damping of turbulence by positive radial density gradient in swirling flow. The inclusion of density fluctuations in the basic conservation equations yielded a modified set of equations for kinetic energy and dissipation of turbulence. The analysis confirms that in density-stratified swirl flow, the ratio of damping of turbulent kinetic energy by density stratification to its generation due to shear stress can be expressed in terms of a dimensionless group, the Modified Richardson number recommended for the characterization of such flows. New equations were used to modify the FLUENT 4.51 source code. Results of CFD calculations made with and without the modified turbulence model were compared with experimental data previously obtained on a ducted turbulent methane diffusion flame in swirling annular air flow. Agreement between experiment and computations greatly improved when the modified turbulence model was used.

1 Nomenclature

$c_{1\varepsilon}$, $c_{2\varepsilon}$	constants
d	nozzle diameter
D	tube diameter
F_j	j^{th} component of the sum of forces
g_j	j^{th} component of gravitational acceleration
G_b	rate of generation of turbulence due to buoyancy
G_k	rate of generation of turbulence due to shear effect
G_s	rate of damping of turbulence due to stratification
k	turbulent kinetic energy (TKE)
p	pressure
r	radial co-ordinate
R_i	Richardson number
R_i^*	Modified Richardson number
R	radius
u_i	i^{th} component of velocity
U	horizontal velocity
T	temperature
w	tangential velocity
x	axial co-ordinate
z	vertical co-ordinate
ε	the dissipation rate of turbulence
μ_ε	$= \mu_t/1.3$
μ_t	turbulent viscosity,
σ_P	turbulent Prandtl number
ρ	density
overbar	means averaging
prime	means fluctuation

2 Introduction

The effect of rotation on mixing and entrainment is greatly influenced by the stability of the system. Rayleigh [1] considered the axially symmetric flow of a rotating fluid without viscosity and in the absence of axial and radial motions. The stability criterion put forward by Rayleigh was that a system is (*I*) stable if ρwr increases with r (solid body rotation) (*ii*) neutrally stable if ρwr is constant with r (free vortex) and (*iii*) unstable if ρwr decreases with r.

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Emmons [2] demonstrated the fire-whirl effect for liquid pool flames formed at the center of a rotating screen and showed that the surrounding air was in a free vortex motion except for a finite-size vortex core in solid-body rotation. In the fire-whirl experiment, the rate of evaporation of fuel was dependent upon the inflow of air in the base boundary layer so that increases in flame length were due to added fuel supply as well as the effect of the rotating environment.

Chigier *et al.*[3] showed experimentally that replacement of the liquid pool by a fuel gas jet in which the supply of fuel is kept constant while the circulation in the surrounding air is increased led to two-fold increases in flame length. Schlieren photographs of flames in the rotating screen showed that the turbulent flame brush became laminarized whereas flame-spread, mixing and entrainment were reduced by rotation of the screen.

Similar flame lengthening could be obtained in burner flames for gaseous, liquid and solid fuels by increasing the swirl number in the annular air flow surrounding the fuel jet as shown by Toqan *et al.*[4], Shihadeh *et al.*[5] and Barta *et al.* [6], respectively.

These experiments demonstrated that increases in flame length could be obtained by the combined effects of positive radial density gradient and rotating flow.

3 Theoretical

In a horizontal flow, if the density decreases with height, the process of turbulent mixing moves heavier fluid layers above the lighter, and *vice versa*. The buoyancy force, however, will act against this process by pushing heavier and lighter layers back to their original locations. Consequently part of the work available to maintain turbulence will be consumed by the act of buoyancy forces, hence, the turbulent motion can diminish or die out. The ratio of work done by the buoyancy force and the kinetic energy of vertical turbulence is called the Richardson number:

$$R_i = -\frac{(g/\rho)(\partial\rho/\partial z)}{(\partial U/\partial z)^2} \quad (1)$$

In a centrifugal flow field with radially increasing density, heavier layers of fluid can be moved radially inwards, and lighter layers outwards, by the action of turbulent mixing. The action of the centrifugal force , however, will hinder such mixing by separating heavier and lighter fluid particles, which process may lead to flow laminarisation. The process of turbulence damping in swirling flow can be characterized by a dimensionless group, the Modified Richardson number as proposed by Beér *et al*. [7]:

$$R_i^* = \frac{(W^2/r\rho)(\partial\rho/\partial r)}{(\partial U/\partial r)^2} \quad (2)$$

In reacting flows, where heat is produced by chemical reaction resulting spatial variations of density, the effect of turbulence damping (or generation) may considerably alter both the flow properties and chemical reaction rates. The standard turbulence models (k-ε or Reynolds Stress Model) can not correctly describe turbulent flows, in which such density stratification occurs. In the following, results are reported of the modification of the k-ε model to enable it to account for turbulence damping in swirl flows with strong density gradients. The present contribution is the continuation of research by Barta *et al*. [8] and Haynes [9].

3.1 Modified k-ε equations

The conservation equations of momentum and mass can be combined to give the following formula:

$$\frac{\partial \rho u_i u_j}{\partial x_i} = -\frac{\partial}{\partial x_i}[-p + \mu(\frac{\partial u_i}{\partial u_j} + \frac{\partial u_j}{\partial x_i})] = F_j \quad (3)$$

The modified k-equation can be obtained in the following way: multiply Eqn.3 by u_j, take the time average and sub-tract the resulting equation from the formula obtained as follows: take the time average of Eqn. 3 and multiply both sides by the time average of u_j. The result is shown below:

$$\frac{\partial}{\partial x_i}\bar{\rho}\,\bar{u}_i\overline{\frac{1}{2}u_j'u_j'} = -\frac{1}{2}\frac{\partial}{\partial x_i}(\bar{u}_i\overline{\rho' u_j' u_j'} + \bar{\rho}\overline{u_i' u_j' u_j'} + \overline{\rho' u_i' u_j' u_j'})$$
$$-(\bar{u}_i\overline{\rho' u_j'} + \bar{\rho}\overline{u_i' u_j'} + \overline{\rho' u_i' u_j'})\frac{\partial \bar{u}_j}{\partial x_i} + \overline{F_j' u_j'} \quad (4)$$

The time average of $F_j u_j$ is given as:

$$\overline{F_j u_j'} = -\overline{u_j'\frac{\partial p'}{\partial x_j}} + \overline{u_j'\frac{\partial}{\partial x_i}[\mu(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i})]} + \overline{\rho' u_j'}g_j \quad (5)$$

Closure approximations are applied after Mohammadi and Pironneau [10]. Convection by random fields produces diffusion for the mean, which can give the following approximation:

$$\frac{1}{2}(\bar{u}_i\overline{\rho' u_j' u_j'} + \bar{\rho}\overline{u_i' u_j' u_j'} + \overline{\rho' u_i' u_j' u_j'}) = -\frac{\mu_t}{\bar{\rho}}\frac{\partial k}{\partial x_i} \quad (6)$$

The velocity-density correlation is calculated as:

$$\overline{\rho' u_j'} = -\frac{\mu_t}{\sigma_P\,\bar{\rho}}\frac{\partial\bar{\rho}}{\partial x_j} \quad (7)$$

The Reynolds stress in the second term of the right hand side of Eqn. 4 is approximated by the known Boussinesque expression, which was used to calculate the rate of generation of turbulent kinetic energy (TKE). The triple correlation in the second expression of the right hand side of Eqn. 4 is neglected. Also, the first term in the right hand side of Eqn. 5 is set to zero. The second term is modeled as fol-lows:

$$\overline{u_j'\frac{\partial}{\partial x_i}[\mu(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i})]} = -\bar{\rho}\varepsilon \quad (8)$$

where *ε*, the rate of turbulence dissipation. The final expression of the k-equation is shown below:

$$\frac{\partial}{\partial x_i}(\bar{\rho}\,\bar{u}_i\,k) = \frac{\partial}{\partial x_i}(\frac{\mu_t}{\bar{\rho}}\frac{\partial k}{\partial x_i}) + G_k - G_s - G_b - \bar{\rho}\varepsilon \quad (9)$$

where

$$G_k = \mu_t(\frac{\partial\bar{u}_i}{\partial x_j} + \frac{\partial\bar{u}_j}{\partial x_i})\frac{\partial\bar{u}_j}{\partial x_i} \quad (10)$$

$$G_s = -\frac{\mu_t}{\sigma_P\,\bar{\rho}}\frac{\partial\bar{\rho}}{\partial x_j}\,\bar{u}_i\frac{\partial\bar{u}_j}{\partial x_i} \quad (11)$$

$$G_b = \frac{\mu_t}{\sigma_P\,\bar{\rho}}\frac{\partial\bar{\rho}}{\partial x_j}g_j \quad (12)$$

Eqn. 10 is represents the rate of generation of TKE due to mean shear. Eqn. 11 represents the new expression to account for the turbulence damping described in experiments referenced in the previous section. The Eqn. 12 is the expression for calculating the rate of turbulence generation (damping) due to buoyancy effect. The similarity between the Eqns. 11 and 12 can be easily seen.

Using the procedure given in reference [10], the rate of turbulence dissipation is calculated by the following differential equation:

$$\bar{\rho}\,\bar{u}_i\frac{\partial\varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}(\mu_\varepsilon\frac{\partial\varepsilon}{\partial x_i}) + c_{1\varepsilon}\frac{\varepsilon}{k}(G_k - G_s - G_b) - c_{2\varepsilon}\bar{\rho}\frac{\varepsilon^2}{k} \quad (13)$$

The final closure is reached by the Eqn. 14:

$$\mu_t = c_\mu\bar{\rho}\frac{k^2}{\varepsilon} \quad (14)$$

As can be seen from Eqns. 9 and 13, the effect of flow stratification in swirling and gravity flow can be characterized by the ratio of $G_s + G_b$ and G_k :

$$R_i^0 = \frac{G_s + G_b}{G_k} \tag{15}$$

It can be shown [8] that, for horizontal flows with vertical density variation, R_i^0 can be reduced to the expression of the Richardson number (Eqn. 1). Also, for strong rotating flows with radial density variation, R_i^0 can be equated with the Modified Richardson number (Eqn. 2).

For the purpose of computational fluid dynamic modeling of an experimental methane flame, the commercially available code, FLUENT 4.51 was modified by the newly derived equations.

The mixture fraction/PDF (Probability Density Function) model was selected to describe the chemical reaction between methane and oxygen. The mixture fraction modeling approach involves the solution of transport equations for one or two conserved scalars (the mixture fractions). The value of mixture fraction at each point in the flow domain is computed by FLUENT 4.51 through solution of the conservation equation for its mean (time averaged) value in the turbulent flow field.

The reacting system is treated using chemical equilibrium calculations. The interaction of turbulence and chemistry is accounted for by a probability density function. The equilibrium assumption implies that the chemistry is rapid enough for chemical equilibrium to always exist at the molecular level.

An algorithm based on the minimization of Gibbs free energy is used to compute species mole fractions from the mixture fraction. The actual species modeled in our example are: CH_4, O_2, CO_2, H_2O, N_2, O, OH, H, H_2 and CO.

The P1 Radiation model was selected to model the radiative heat transfer. The P1 model is based on the expansion of the radiation intensity into an orthogonal series of spherical harmonics. If only four terms in the series are used an adequate set of equations can be obtained for the calculation of the radiative flux .

Details of the mixture fraction/PDF and P1 radiation model can be found in the references [11], and [12] respectively.

4 Experimental

To test the modified turbulence model a set of experimental data obtained earlier with a methane jet burning along the axis of a rotating annular air flow (Beér *et al.* [7]) was compared with model predictions. The experimental apparatus consisted of a cylindrical tube mounted on a variable swirl generator.

Adjustment of movable blocks varied the proportion of air introduced tangentially and, thus, the degree of rotation imparted to the air could be varied while maintaining constant flow rates.

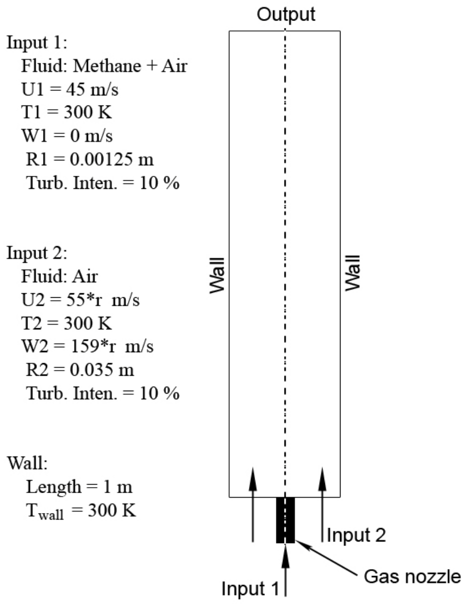

Fig. 1 Experimental input conditions [7]

The experimental conditions are summarized in Fig. 1. The tube diameter was 70 mm. In the center, a gas nozzle of 2.5 mm diameter was placed to produce a partially pre-mixed methane/primary air jet of 45 m s^{-1} axial velocity. The volumetric fraction of primary air in the methane-air mixture was 36.2%.

The secondary air was supplied with high swirl (Swirl number**:** 2) at 300 1 min^{-1}. The air temperature was 300 K. Measurements of axial and tangential velocity near the inlet indicated a fluid flow of solid body rotation. The inlet velocity profiles for axial and tangential velocity were set according to measured values. The wall temperature was constant at 300 K. Flame temperatures were measured with a Pt/Pt-Rh thermocouple.

Beér *et al.* [7] found that the visible flame length increased with increasing swirl in the secondary air flow. At the modeled experimental condition, the flame reached a length of 320 d. Flames were highly luminous with large quantities of soot, and measured oxygen concentrations were reduced within the flame. These temperature and concentration measurements were evidence of the reduced entrainment of air into the flame with increasing degree of swirl.

In a co-axial swirling flow where the inner flow is of high temperature and the outer is cold, a radially stratified flow can develop under certain conditions.

In swirling flow, the Eqn. 15 expressed in cylindrical coordinates, may be reduced to the following form with neglecting small terms:

$$R_i^0 = \frac{\dfrac{\overline{w}^2}{\sigma_P \, r \, \overline{\rho}} \dfrac{\partial \overline{\rho}}{\partial r}}{\left(\dfrac{\partial u}{\partial r}\right)^2 + \left(\dfrac{\partial w}{\partial r}\right)^2} \tag{16}$$

When analyzing the R_i^0 number, one can conclude that in a co-axial arrangement of high-temperature inner, and cold-swirling outer flow, high degree of radial stratification can be reached ($R_i^0 >> 1$).

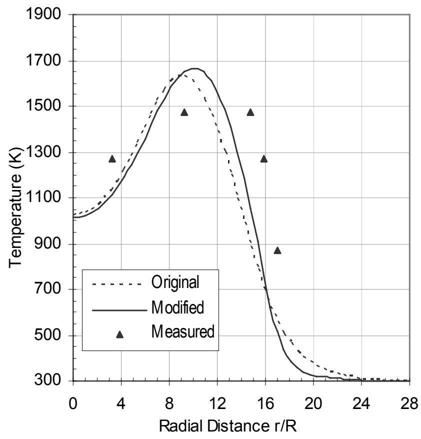

Fig. 2 Radial profile of temperature at x/D=29

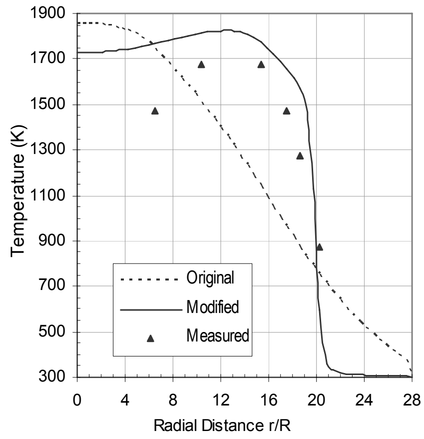

Fig. 4 Radial profile of temperature at x/D=79

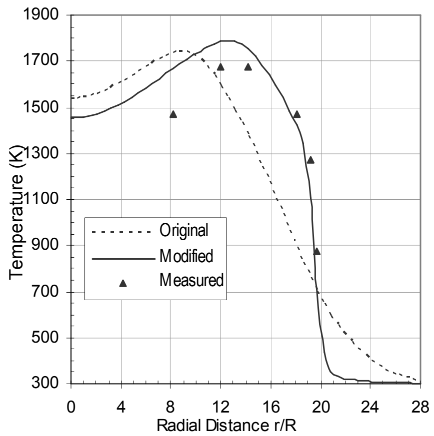

Fig. 3 Radial profile of temperature at x/D=54

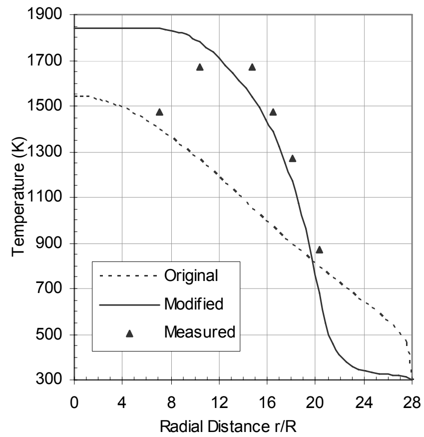

Fig. 5 Radial profile of temperature at x/D=104

From the results of the CFD calculations, radial temperature profiles taken at the axial locations of x/D = 29, 54, 79 and 104 are shown in Figs. 2, 3, 4 and 5, respectively. Three sets of data are shown. The dotted line indicates the results calculated by the original FLUENT 4.51. code, in which the k-ε model was not modified. The triangles are experimental results and the solid line represents the modified FLUENT 4.51 model calculations.

As seen in Figs. 2., 3., 4. and 5., the calculated temperatures obtained by the modified k-ε model matched the experimental data with good accuracy, whereas the original k-ε model broke down. The original k-ε model calculations yield a flat temperature profile, especially for the axial distance of x/D=104, in contrast to the modified code which can capture the measured steep temperature gradient near the wall created by the reduced rate of radial mixing of air towards the center of the flame.

The agreement, however, is less satisfactory in either case near the axis, which may be due either to the failing of the chemical reaction mechanism model or error in the temperature measurement in the strongly sooting portion of the flame.

Isotherms within the flames calculated by the original and the modified k-ε turbulence model show the changes in flame shape. As seen in Fig. 6, in the case of modified model, a longer flame length was predicted with the modified k-ε in agreement with the experimental findings. As expected from the reduced radial mixing of air, the temperature gradient at the outer edge of the flame is higher then calculated with the modified turbulence model.

In Fig. 7, the maps of the TKE are shown for the two model cases. Due to the action of stratification modeled by the turbulence damping term, G_s, the TKE at the flame edges is smaller than that calculated by the original k-ε model. Accordingly, as can be seen in Fig. 8, the flame edge marks the region where R_i^0 calculated from the Eqn. 16 reaches its maximum value.

5 Conclusions

Based on theoretical considerations, the k-ε model of turbulence was modified to include the effects of density fluctuations on turbulence. Closure approximations were used to simplify the resultant form.

The use of the modified turbulence model produced changes in the calculated velocity, species concentration, and temperature fields of turbulent reacting flows.

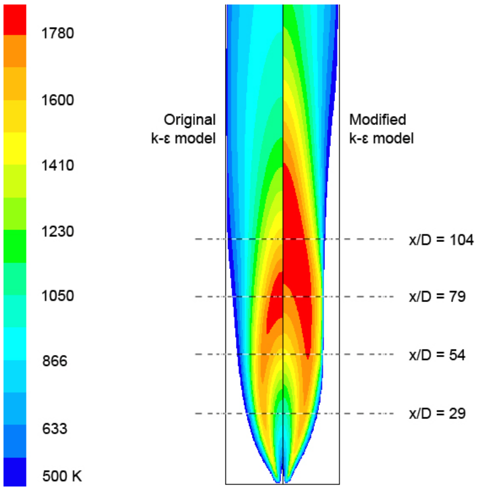

Fig. 6 Comparison of flame temperature distributions computed by the original and the modified k-ε model

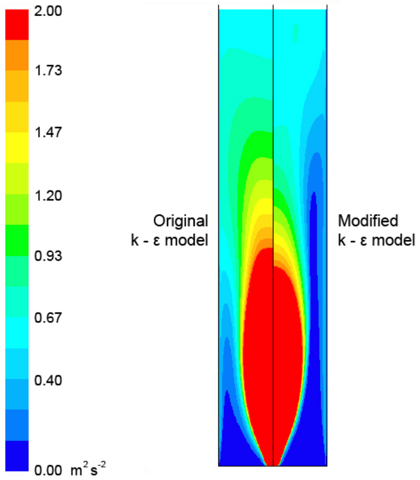

Fig. 7 Comparison of Turbulent Kinetic Energy distributions computed by the original and the modified k-ε model

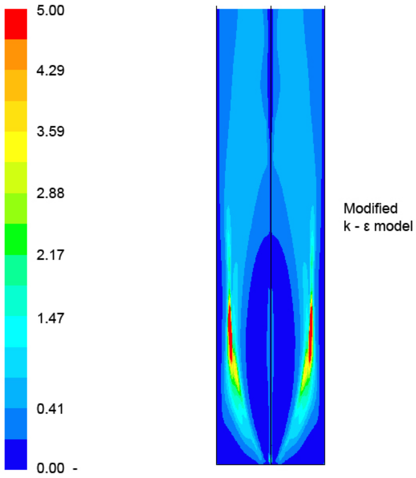

Fig. 8 Distribution of the Richardson number calculated by Eqn. 16 in the modified k-ε model

The new equation was used to modify the FLUENT 4.51 code, and test calculations made with and without the modified turbulence model showed significant differences in the flame structure.

Experimental data obtained earlier on a turbulent methane flame surrounded by swirling airflow have then been chosen for comparison with calculations made by means of the unaltered and the modified FLUENT code, respectively. The code with the modified k-ε turbulence model was found to be capable of predicting the experimentally determined changes in flame structure, especially the lengthening of the flame due to the combination of the positive radial density gradient and the rotating air flow surrounding the flame.

Comparisons of experimental data and calculations showed that the modified turbulence model enabled the CFD code to predict correctly flows in inertial force fields with density variations, cases in which the application of the unaltered code was not successful.

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